सन्मार्ग मेधा

मॉक प्रश्न पत्र

उच्च माध्यमिक

विषय : गणित
कक्षा : १२

General Instructions:

(i) This question paper contains 38 questions. All questions are compulsory.
(ii) This question paper is divided into five Sections A, B, C, D and E.
(iii) In Section A, Questions no. 1 to 18 are multiple choice questions (MCQs) and questions number 19 and 20 are Assertion-Reason based questions of 1 mark each.
(iv) In Section B, Questions no. 21 to 25 are very short answer (VSA) type questions, carrying 2 marks each.
(v) In Section C, Questions no. 26 to 31 are short answer (SA) type questions, carrying 3 marks each.
(vi) In Section D, Questions no. 32 to 35 are long answer (LA) type questions carrying 5 marks each.
(vii) In Section E, Questions no. 36 to 38 are case study based questions carrying 4 marks each.
(viii) Use of calculators is not allowed.

SECTION A

This section comprises multiple choice questions (MCQs) of 1 mark each.

1. If $A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}$ and $A^2 - xA = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then the value of x is [1]

(A) – 4 (B) 4 (C) – 2 (D) 2

2. If A is a square matrix of order 3 such that [1]
$A.(adj\ A) = \begin{bmatrix} -2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{bmatrix}$, then $|adj\ A|$ is equal to

(A) – 2 (B) – 4 (C) 4 (D) – 8

3. If the area of the triangle with vertices (3, 2), (– 1, 4) and (6, k) is 7 sq. units, then possible values of k is/are [1]

(A) 3 (B) – 3 , 4 (C) – 4 (D) 3, – 4

4. $\int_{-1}^{1} |x-1|\, dx$ is equal to

(A) 1 (B) 2 (C) 3 (D) -3 [1]

5. The angle between the lines $\frac{x+4}{1} = \frac{y-3}{2} = \frac{z+2}{3}$ and $\frac{x}{3} = \frac{y}{-2} = \frac{z}{1}$ is [1]

(A) $\sin^{-1}\frac{1}{7}$ (B) $\cos^{-1}\frac{1}{7}$ (C) $\cos^{-1}\frac{2}{7}$ (D) $\sin^{-1}\frac{2}{7}$

6. Integrating factor of the differential equation $\cos x \frac{dy}{dx} + y \sin x = 1$ is [1]

(A) $\cos x$ (B) $\sin x$ (C) $\tan x$ (D) $\sec x$

7. In an L.P.P. if the objective function $Z = ax + by$ has the same maximum value on two corner points of the feasible region, then the number of points at which maximum value of Z occurs is [1]

(A) 0 (B) 2 (C) finite (D) infinite

8. If $|\vec{a}| = 2$, $|\vec{b}| = 7$ and $\vec{a} \times \vec{b} = 3\hat{\imath} + 2\hat{\jmath} - 6\hat{k}$, then the angle between $\vec{a}$ and $\vec{b}$ is [1]

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{3}$ (D) $\frac{2\pi}{3}$

9. If $f(x) = \begin{cases} \frac{\sin(e^{x-2}-1)}{\log(x-1)}, & x \neq 2 \\ k, & x = 2 \end{cases}$ then the value of k for which $f(x)$ is continuous is [1]

(A) – 2 (B) – 1 (C) 0 (D) 1

10. The number of possible reflexive relations on a set consisting of 3 elements is [1]

(A) 512 (B) 64 (C) 256 (D) 128

11. The principle value of $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ is [1]

(A) $-\frac{2\pi}{3}$ (B) $-\frac{\pi}{3}$ (C) $-\frac{\pi}{6}$ (D) $\frac{2\pi}{3}$

12. The area of the triangle formed by the vertices O , A and B where $\overrightarrow{OA} = \hat{\imath} + 2\hat{\jmath} + 3\hat{k}$ and $\overrightarrow{OB} = -3\hat{\imath} - 2\hat{\jmath} + \hat{k}$ is (in sq. units) [1]

(A) $5\sqrt{5}$ (B) $3\sqrt{5}$ (C) $6\sqrt{5}$ (D) 4

13. For the curve $\sqrt{x} + \sqrt{y} = 1$, $\frac{dy}{dx}$ at $\left(\frac{1}{4}, \frac{1}{4}\right)$ is [1]

(A) $\frac{1}{2}$ (B) – 1 (C) 1 (D) 2

14. A bag contains 5 red and 3 blue balls. If 3 balls are drawn at random without replacement, then the probability that exactly two of these balls were red, the first ball being red, is [1]

(A) $\frac{5}{7}$ (B) $\frac{4}{7}$ (C) $\frac{15}{28}$ (D) $\frac{5}{14}$

15. If p and q are the degree and order of the differential equation $\left(\frac{d^2y}{dx^2}\right)^2 + 3\frac{dy}{dx} + \frac{d^3y}{dx^3} = 4$, then the value of $2p - 3q$ is [1]

(A) 7 (B) – 7 (C) 3 (D) – 3

16. The value of p for which $\vec{a} = 3\hat{\imath} + 2\hat{\jmath} + 9\hat{k}$ and $\vec{b} = \hat{\imath} + p\hat{\jmath} + 3\hat{k}$ are parallel vectors is [1]

(A) 3 (B) $\frac{3}{2}$ (C) $\frac{2}{3}$ (D) $\frac{1}{3}$

17. If $f(x^3) = x^5$ for all $x \in R,\ x \neq 0$. Then the value of $f'(8)$ is [1]

(A) 20 (B) $\frac{20}{3}$ (C) $\frac{5}{3}$ (D) 80

18. If the lines $6x - 2 = 3y + 1 = 2z - 2$ and $\frac{x-2}{\lambda} = \frac{2y-5}{-3} = \frac{z+2}{0}$ are perpendicular, then $\lambda =$ [1]

(A) 3 (B) 2 (C) – 3 (D) 1

Questions number 19 and 20 are Assertion and Reason based questions carrying 1 mark each. Two statements are given, one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer from the codes (a), (b), (c) and (d) as given below.

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A).
(b) Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of the Assertion (A).
(c) Assertion (A) is true and Reason (R) is false.
(d) Assertion (A) is false and Reason (R) is true.

19. Assertion (A) : The vector equation of the line passing through the points (6,-4,5) and (3,4,1) is $\vec{r} = (6\hat{\imath} - 4\hat{\jmath} + 5\hat{k}) + \lambda(-\hat{3}i + 8\hat{\jmath} + 4\hat{k})$. [1]

Reason (R) : The vector equation of the line passing through the points $\vec{a}$ and $\vec{b}$ is $\vec{r} = \vec{a} + \lambda(\vec{b} - \vec{a})$

20. Assertion (A) : The relation $f: \{1, 2, 3, 4\} \to \{x, y, y, z, p\}$ defined by $f = \{(1, x), (2, y), (3, z)\}$ is a bijective function.

Reason (R) : The function $f: \{1, 2, 3\} \to \{x, y, y, z, p\}$ Such that $f = \{(1, x), (2, y), (3, z)\}$ is one-one.

SECTION B

This section comprises very short answer (VSA) type questions of 2 marks each.

21. A balloon which always remains spherical is being inflated by pumping in 900 cubic centimetres of gas per second. Find the rate at which the radius of the balloon is increasing when its radius is 15 cm. [2]

22. Find the domain of the function $\cos^{-1}(2x-3)$ [2]

OR,

Find the value of $\tan^{-1}(-\sqrt{3}) + \cos^{-1}\left(-\frac{1}{2}\right) - \mathrm{cosec}^{-1}(\sqrt{2})$

23. Manufacturer can sell x items at a price of Rs $\left(5 - \frac{x}{100}\right)$ each. [2]

The cost price of x items is Rs. $\left(\frac{x}{5} + 500\right)$.

Find the number of items he should sell to earn maximum profit.

24. Find a unit vector perpendicular to each of the vectors $(\vec{a} + \vec{b})$ and $(\vec{a} - \vec{b})$, where $\vec{a} = 3\hat{i} + 2\hat{j} + 2\hat{k}$ and $\vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$. [2]

25. If $y = (\sin^{-1} x)^2$, prove that $(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - 2 = 0$. [2]

SECTION C

This section comprises short answer (SA) type questions of 3 marks each.

26. Find $\int \frac{2}{(1-x)(1+x^2)}\,dx$ [3]

27. A and B are two independent events. The probability that both A and B occur is $\frac{1}{6}$ and the probability that neither of them occur is $\frac{1}{3}$. [3]

Find the probability of the occurrence of A.

28. Evaluate: $\int_0^1 \frac{\log(1+x)}{1+x^2}\,dx$ [3]

29. Solve the following Linear Programming Problem graphically: [3]

Minimize: $z = 3x + 9y$,

subject to the constraints:

$x + y \geq 10$, $x + 3y \leq 60$, $x \leq y$, $x \geq 0$, $y \geq 0$.

30. If $x\sqrt{1+y} + y\sqrt{1+x} = 0$, for $-1 < x < 1$, show that $\frac{dy}{dx} = -\frac{1}{(1+x)^2}$ [3]

31. Prove that the function $f(x) = \log(1 + x) - \frac{2x}{x+2}$ is increasing throughout its domain. [3]

SECTION D

This section comprises long answer type questions (LA) of 5 marks each.

32. If $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$, find AB. [5]

Hence, solve the following system of equations

$x - y = 3;\quad 2x + 3y + 4z = 17;\quad y + 2z = 7.$

33. Using integration find the area of the region bounded by the line $x + y + 2 = 0$ and the curve $x^2 = -y$. [5]

34. Let $A = \{1, 2, 3, \ldots, 9\}$ and R be the relation defined on $A \times A$ by $(a, b)R(c, d)$ iff $a + d = b + c$. [5]

Prove that R is an equivalence relation. Also find the equivalence class $[(2, 5)]$.

OR,

Show that the function $f: R \to R$, defined by $f(x) = 2x^3 - 7$ for all $x \in R$, is one − one and onto function.

35. Find the shortest distance between the lines [5]

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4} \quad \text{and} \quad \frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$$

OR,

Show that the lines $\frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7}$ and $\frac{x-2}{1} = \frac{y-4}{3} = \frac{z-6}{5}$ intersect.

Also find the coordinates of their point of intersection.

SECTION E

This section comprises case study based questions of 4 marks each.

36. A toy making company made a toy in which a cylinder is inscribed in a sphere of radius R. The height and radius of the cylinder is h and r, respectively. [4]

On the basis of above information, answer the following questions:

(i) Find the relation between h and r.

(ii) Find the value of volume of cylinder V in terms of h.

(iii) Find the value of h when V is maximum.

OR,

(iii) If V is maximum at $h = \frac{2}{\sqrt{3}}R,$ then find the maximum value of V.

37. A farmer moves along the boundary of a triangular field ABC. Three vertices of the triangular field are A (1, 2, 3), B (-1, 0, 0) and C (0, 1, 2) respectively. [4]

On the basis of above information, answer the following questions:

(i) Find the area of the triangle ABC.

(ii) Find the projection of $\overrightarrow{BA}$ on $\overrightarrow{BC}$.

(iii) Find $\angle ABC$.

38. A random variable X has the following probability distribution [4]

x	0	1	2	3	4	5	6	7	8
P(X)	a	3a	5a	7a	9a	11a	13a	15a	17a

On the basis of above information, answer the following.

(i) Find the value of a

(ii) Find P(X=4).

(iii) Find $P(0 \leq X \leq 2)$.