सन्मार्ग मेधा

मॉक प्रश्न पत्र

उच्च माध्यमिक

विषय : गणित
कक्षा : १२

TIME: 3 HOURS **TOTAL MARKS: 80**

General Instructions:

1. **This Question paper contains five sections - A, B, C, D and E. Each section is compulsory.**
2. **Section A has 18 MCQs and 02 Assertion-Reason (A-R) based questions of 1 mark each.**
   Section B has 05 questions of 2 marks each.
   Section C has 06 questions of 3 marks each.
   Section D has 04 questions of 5 marks each.
   Section E has 03 Case-study questions with sub-parts (4 marks each).

SECTION A

QUESTIONS 1 TO 10 CARRY 1 MARK EACH

1. The function $f(x) = \frac{3-x}{3x-x^2}$ is discontinuous at — 1
   a) only one point
   b) exactly two points
   c) exactly thee points
   d) None of these

2. If there are two values of 'a' which makes determinant, — 1
   $\Delta = \begin{vmatrix} 1 & -2 & 5 \\ 2 & a & -1 \\ 0 & 4 & 2a \end{vmatrix} = 86$ then the sum of these values is

   a) 4 b) 5 c) -4 d) 9

3. A relation R in the set of real numbers **R** is defined as aRb if $a > b$. Then R is — 1
   a) an equivalence relation
   b) transitive but neither reflexive nor symmetric
   c) symmetric ,transitive but not reflexive
   d) neither transitive nor reflexive but symmetric

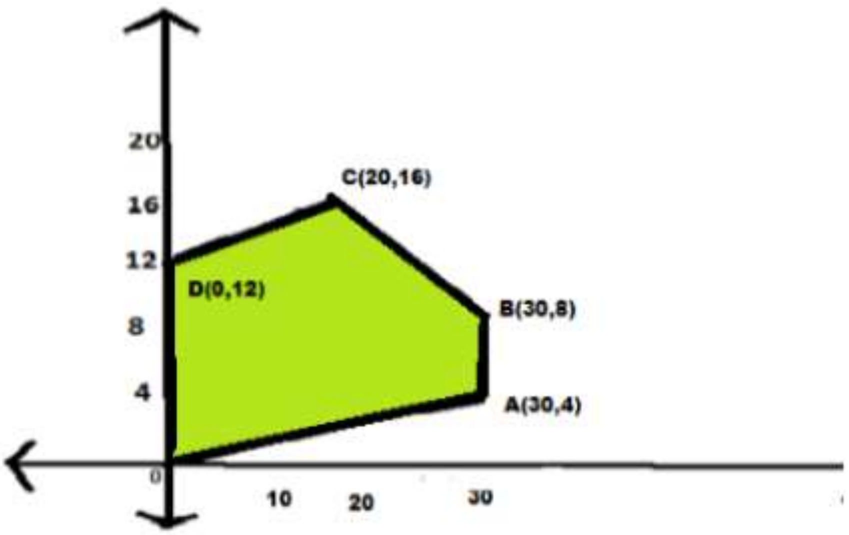
4. $\tan^{-1}\sqrt{3} - \sec^{-1}(-2) =$ 1

a) $-\pi$ b) $\frac{\pi}{2}$ c) $-\frac{\pi}{3}$ d) $\frac{2\pi}{3}$

5. The number of all possible matrices of order 2×2 with each entry either 1 or 2 or 3 or 4 is 1

a) 16 b) 64 c) 256 d)1024

6. The corner points of the feasible region determined by the system of linear constraints are as shown below: 1



Let $Z = px + qy$ where $p, q > 0$ be the objective function. Then the condition on p and q so that the maximum value of Z occurs at B(30,8) and C(20,16) is

a) $4p = 3q$ b)$3p = 4q$ c) $5p = 4q$ d) $2p = q$

7. The maximum or minimum values,if any, for $f(x) = -|x + 4| + 6$ are 1

a) maximum 6 and no minimum value
b) minimum 6 and no maximum value
c) miimum 0 and maximum 6
d) None of the above

8. The function $f(x) = \tan^{-1}(\text{sinx} + \text{cosx})$ in the interval (0 , π/4) is 1

a) strictly increasing function
b) strictly decreasing function
c) an increasing function
d) neither increasing nor decreasing

9. $If\ \int \frac{x^3}{\sqrt{1+x^2}}dx = a(1+x^2)^{3/2} + b\left(\sqrt{1+x^2}\ \right) + C\ then$ 1

$a)\ a = \frac{1}{3}, b = 1$ b) $a = -\frac{1}{3}, b = 1$

c) $a = -\frac{1}{3}, b = -1$ d) $a = \frac{1}{3}, b = -1$

10. The area of the region bounded by the curve $y = \sqrt{16 - x^2}\ and\ x\ axis\ is$ 1

a) $8\pi\ sq\ units$ b) $20\pi\ sq\ units$
c) $16\pi\ sq\ units$ **d) 256**$\pi\ sq\ units$

11. The degree of the differential equation $[1 + \left(\frac{dy}{dx}\right)^2]^{3/2} = \frac{d^2y}{dx^2}$ is 1

a) 1 b) 2 c) 3 d) 4

12. If $\vec{p}$ is a unit vector and $(\vec{x} - \vec{p})(\vec{x} + \vec{p}) = 80$, then the value of $|\vec{x}|$ is 1

a) 6 b) 7 c) 8 d) 9

13. If $3\tan^{-1} x + \cot^{-1} x = \pi$ then $x =$ 1

a)0 b) 1 c)-1 d) $\frac{1}{2}$

14. The position vector of the point which divides the join of points $2\vec{a} - 3\vec{b}$ and $\vec{a} + \vec{b}$ in the ratio 3: 1 is 1

a)$\frac{3\vec{a}-2\vec{b}}{2}$ b) $\frac{7\vec{a}-8\vec{b}}{4}$

c) $\frac{3\vec{a}}{4}$ d) $\frac{5\vec{a}}{4}$

15. If $A = [a_{ij}]$ is a skew symmetric matrix of order n then 1

a) $a_{ij} = \frac{1}{a_{ji}} \forall i, j$ b) $a_{ij} \neq 0\ \forall\ i, j$

c) a_{ij} = 0 $\forall i = j$ d) $a_{ij} \neq 0\ \forall i = j$

16. The direction cosines of vector $2\hat{\imath} + 2\hat{\jmath} - \hat{k}$ are 1

a)$\frac{2}{5}, \frac{2}{5}, \frac{1}{5}$ b) $\frac{2}{5}, \frac{2}{5}, \frac{-1}{5}$ c) $\frac{2}{3}, \frac{2}{3}, \frac{1}{3}$ d) $\frac{2}{3}, \frac{2}{3}, \frac{-1}{3}$

17. The distance of the point (a,b,c) from y axis is **1**

a) b b) $|b|$ c) $|b| + |c|$ d)$\sqrt{a^2 + c^2}$

18. Three persons A, B and C fire at a target in turns, starting with A . Their probabilities of hitting the target are 0.4 , 0.3 and 0.2 respectively. The probabilities of exactly two hits is 1

a) 0.024 b) 0.188 c) 0.336 d) 0.452

In question numbers 19 and 20 a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct option out of the following choices.

a) Both assertion (A) and reason (R) are true, and reason (R) is the correct explanation of assertion (A).

b) Both assertion (A) and reason (R) are true, and reason (R) is not the correct explanation of assertion (A).

c) Assertion (A) is true, but reason (R) is false.

d) Assertion (A) is false, but reason (R) is true.

19. Assertion (A) : The area of the region bounded by the curve $y = x + 1$ and the lines $x = 2$ and $x = 3$ is $\frac{9}{2}$ square units. 1

Reason (R) : Required area is given by $\int_2^3 (x + 1)dx$

20. Assertion (A) : The solution of $\frac{dy}{dx} - y = 1$, $y(0) = 1$ is given by $y = 2e^x - 1$ 1

Reason (R) : The given differential equation can be solved by variable separable method.

SECTION B

(Question numbers 21 to 25 carry 2 marks each.)

21. Express the matrix $A = \begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix}$ as the sum of symmetric and skew symmetric matrix. 2

22. Check whether the function f: R → R defined as $f(x) = x^3$ is one-one or not. 2

OR

An equivalence relation R in A divides it into equivalence classes A1, A2, A3. What is the value of A1 ∪ A2 ∪ A3 and A1 ∩ A2 ∩ A3?

23. Find $\int \frac{dx}{x(6(logx)^2+7logx+2)}$ 2

24. Find $\int (\frac{1-x}{1+x^2})^2 \, e^x \, dx$ 2

OR

Find $\int \frac{(x-3)e^x}{(x-1)^3}\, dx$

25. If $|\vec{a}| = 10$, $|\vec{b}| = 2$ and $\vec{a}.\vec{b} = 12$, then find the value of $|\vec{a} \times \vec{b}|$ 2

SECTION C
(Question numbers 26 to 31 carry 3 marks each.)

26. Solve the differential equation : $\frac{dy}{dx} + y\, cotx = 2x + x^2 cotx$, $x \neq 0$ 3

OR

Solve the differential equation: $x\left(\frac{dy}{dx}\right) = y - xtan(\frac{y}{x})$

27. If $x = asin2t(1 + cos2t) and\ y = bcos2t(1 - cos2t)$, $show\ that$ 3

$$(\frac{dy}{dx})_{t=\frac{\pi}{4}} = \frac{b}{a}$$

28. If $y = \frac{logx}{x}$, $show\ that\ \frac{d^2y}{dx^2} = \frac{2logx-3}{x^3}$ 3

OR

Examine the differentiability of the function

$f(x) = \begin{cases} x[x], if\ 0 \leq x < 2 \\ (x-1)x, if\ 2 \leq x < 3 \end{cases}$ at x=2

29. Evaluate $\int_{-5}^{5} |x + 2|\, dx$ 3

30. Show that the relation R in the set A = {1,2,3,4,5} given by 3
R = {(a, b) : |a − b| is divisible by 2; a, b ∈ A} is an equivalence relation.

31. A couple has two children. Find the probability that both are boys if it is known that 3
   i) at least one of them is a boy.
   ii) the older child is a boy.

OR

In a meeting, 70% of the members favour and 30% oppose a certain proposal. A member is selected at random and we take X=0 if he opposes and X=1 if he is in favour. Find E(X).

SECTION D

(Question numbers 32 to 35 carry 5 marks each)

32. Using integration, find the area bounded by the lines 5

$$x + 2y = 2\ ;\ y - x = 1\ and\ 2x + y = 7$$

OR

Using integration , find the region in the first quadrant enclosed by the y axis, the line $y = x$ and the circle $x^2 + y^2 = 32$

33. If $A = \begin{bmatrix} 3 & 2 & -1 \\ -2 & 1 & 2 \\ 1 & -3 & 1 \end{bmatrix}$ find A^{-1}. Hence , solve the system of linear equations: 5

$$3x - 2y + z = 2$$
$$2x + y - 3z = -5$$
$$-x + 2y + z = 6$$

34. Show that the lines : $\frac{x+1}{3} = \frac{y+3}{5} = \frac{z+5}{7}$ and $\frac{x-2}{1} = \frac{y-4}{3} = \frac{z-6}{5}$ intersect. Also, find the point of intersection. 5

OR

Find the shortest distance between the lines :
$\vec{r} = \left(2\hat{j} - 3\hat{k}\right) + \lambda(\hat{\imath} + 2\hat{j} + 3\widehat{k)}\ and\ \vec{r} = 2\hat{\imath} + 6\hat{j} + 3\hat{k} + \mu(2\hat{\imath} + 3\hat{j} + 4\hat{k})$
and also ,write the nature of the lines and justify your answer.

35. Solve the following LPP graphically: 5

Minimize $Z = 4x + 6y$
Subject to constraints
$3x + 6y \geq 80$
$4x + 3y \geq 100$
$x, y \geq 0$

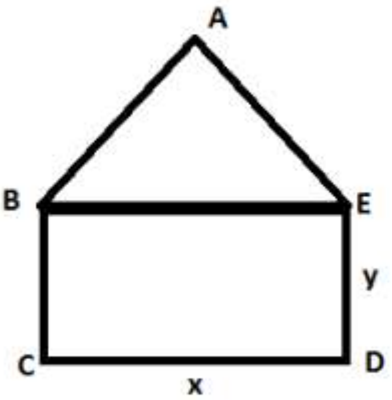
SECTION E

(Question numbers 36 to 38 carry 4 marks each.)

This section contains three Case-study based questions.

The first two questions have three sub-parts (i), (ii) and (iii) of marks 1, 1 and 2 respectively . The third question has two sub-parts of 2 marks each.

36. An architect has designed a window of a house in the shape of a rectangle surmounted by an equilateral triangle. The perimeter of the whole window is 12 m. The owner of the house wants the maximum amount of sunlight through it. 1+1+2



Based on the above information answer the following:

i. If x and y represents the length and breadth of the rectangular region, then find the relation between the variables x and y

ii. Find the area of the window expressed as a function of x

iii. Find the value of y for maximum sunlight

OR

Find the critical point.

37. A factory has two machines A and B. Past record shows that machine A produced 60% of the items and machine B produced 40% of the items. Further, 2 % of the items produced by machine A and 1% produced by machine B were defective. All the items are put into one stock file and then one item is chosen at random. 1+1+2

Based on the above information answer the following:

i. Find the conditional probability that the item is defective given that it is produced by machine B .

ii. Find the total probability of finding the item to be defective.

iii. Find the probability that the item is produced by machine B given that it is defective .

OR

Find the probability that the item is produced by machine A given that it is defective .

38. The equation of motion of a missile are x = 3t, y = -4t, z = t, where the time 't' is given in seconds, and the distance is measured in kilometres. 2+2

Based on the above information answer the following:

i. Which of the following points lie on the path of the missile ?

(6, 8, 2) ; (6, -8, -2) ; (6, -8, 2) ; (-6, -8, 2)

Justify your answer.

ii. At what distance will the rocket be from the starting point (0, 0, 0) in 5 seconds?

अनु खोसला

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